EXAMPLE QUESTIONS

1. Express the following fraction as a sum of its partial fractions.

$$\frac{4x^2 + 2}{x^3 - 4x^2 + 4x + 2x^2 - 8x + 8}$$

2. Convert the following equation from polar to Cartesian coordinates.

$$4r^2 + r^3 \cos \theta - 3 = r \sin \theta$$

3. The speed of a formula 1 car is given by $s = 103 - t^5 e^{-t}$, where t is time, what is the maximum speed of the car? And at what time does this maximum occur?

4. Evaluate the following integral

$$\int_{x=1}^{3} \int_{y=1}^{4x} xy^2 - 3x^2 + 4y \ dydx$$

5. Given $\mathbf{F} = y^2 \hat{\mathbf{i}} - 2x \hat{\mathbf{j}} + xy \hat{\mathbf{k}}$ and $f = x^2 y + z^3$, compute the following.

   (a) $\triangledown \cdot \mathbf{F}$
   (b) $\triangledown \times \mathbf{F}$
   (c) $\triangledown f$

6. Find the general solution to the following ODE.

$$\frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} - 14y = 10e^{3x}$$

7. We have $f = x^3 y + z^2 y - 2x^2 z$, find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial z}$.

8. Let z be a complex number where $\theta = \frac{\pi}{7}$ and $r = 2$. Compute z^7.

9. Find the vector equation of a line that passes through (2, 5, 1) and (7, 4, 3).
   Give your answer in Cartesian form.

10. Let $A = \begin{pmatrix} 2 & 1 \\ -6 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ -5 & 9 \end{pmatrix}$. Compute the following:

    (a) $3A - B$
    (b) AB
    (c) B^{-1}
    (d) The eigenvalues of A and B.

11. Let $f(x) = x^3 - 5x + 2$.

    (a) Show that a root exists between $x = -2$ and $x = -3$.
    (b) Starting with $x_1 = -2$, use the Newton-Raphson method to compute x_2 and x_3.
        Your answers should be to 3 decimal places.

12. Three events A, B and C have the following probabilities: $P(A) = 0.4$, $P(B) = 0.5$, $P(C) = 0.5$, $P(A \cap B) = 0.15$, $P(A \cap C) = 0.2$, $P(B \cap C) = 0.2$ and $P(A \cap B \cap C) = 0.1$. Compute the following:

   (a) $P(A \cup B)$

   (b) $P(A \cup B \cup C)$

   (c) $P(A \cup C')$

   (d) $P(A \cup B | C)$

13. (a) What is the difference between a Type I error and a Type II error?

   (b) A company claims that the paper it manufactures has weight 50 grams per square metre (gsm). To test the company claim, 16 sheets of paper were weighed and their mean weight was $\bar{x} = 49.95$ gsm. Assume that the 16 weights can be modelled as independent and identically distributed random variables with a Normal distribution and standard deviation $\sigma = 0.1$ gsm. Conduct a test of size 5% of the null hypothesis that the expected weight of the paper is 50 gsm against the alternative hypothesis that the expected weight of the paper is not 50 gsm. Hint: Your critical value for a two-tailed test of size 5% is $c = 1.96$.

   Remember to clearly state your test statistic, its null distribution, the critical region, and your conclusion about the companys claim.